CMSC201
Computer Science I for Majors

Lecture 20 – Recursion

Last Class We Covered

- Python’s standard library
- Importing modules

- “Random” numbers
  - Pseudo randomness
  - Seeding the RNG
  - Generating random numbers/choices
    - Three different methods

Any Questions from Last Time?

Today's Objectives

- To introduce recursion
- To better understand the concept of "stacks"
- To begin to learn how to "think recursively"
  - To look at examples of recursive code
  - Summation, factorial, etc.

Introduction to Recursion

What is Recursion?

- In computer science, ***recursion*** is a way of thinking about and solving problems
- It’s actually one of the central ideas of CS

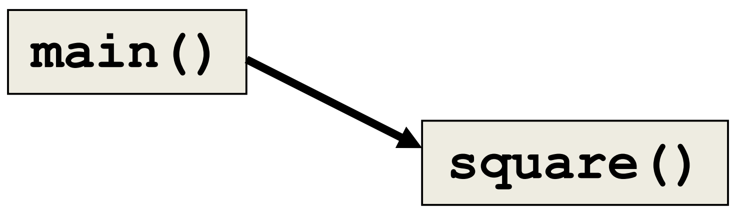
- In recursion, the solution depends on solutions to smaller instances of the <u>same</u> problem

Recursive Solutions

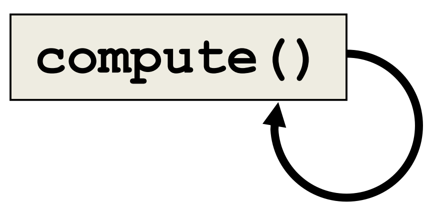
- When creating a recursive solution, there are a few things we want to keep in mind:
  1. We need to break the problem into smaller pieces of itself
  2. We need to define a “base case” to stop at
  3. The smaller problems we break down into need to eventually reach the base case

Normal vs Recursive Functions

- So far, we've had functions call other functions
  - For example, **`main()`** calls the **`square()`** function



- A recursive function, however, calls itself



Why Would We Use Recursion?

- In computer science, some problems are more easily solved by using recursive methods
- For example:
  - Traversing through a directory or file system
  - Traversing through a tree of search results
  - Some sorting algorithms recursively sort data
- For today, we will focus on the basic structure of using recursive methods

Toy Example of Recursion

```
def compute(intInput):
    print(intInput)
    if (intInput > 2):
        compute(intInput-1)

def main():
    compute(50)

main()
```

This is where the recursion occurs.

You can see that the **compute()** function calls itself.

What does this program do?

This program prints the numbers from 50 down to 2.

Visualizing Recursion

- To understand how recursion works, it helps to visualize what’s going on.
- Python uses a ***stack*** to keep track of function calls
- A stack is an important computer science concept

Stacks

 Image from www.topwithcinnamon.com

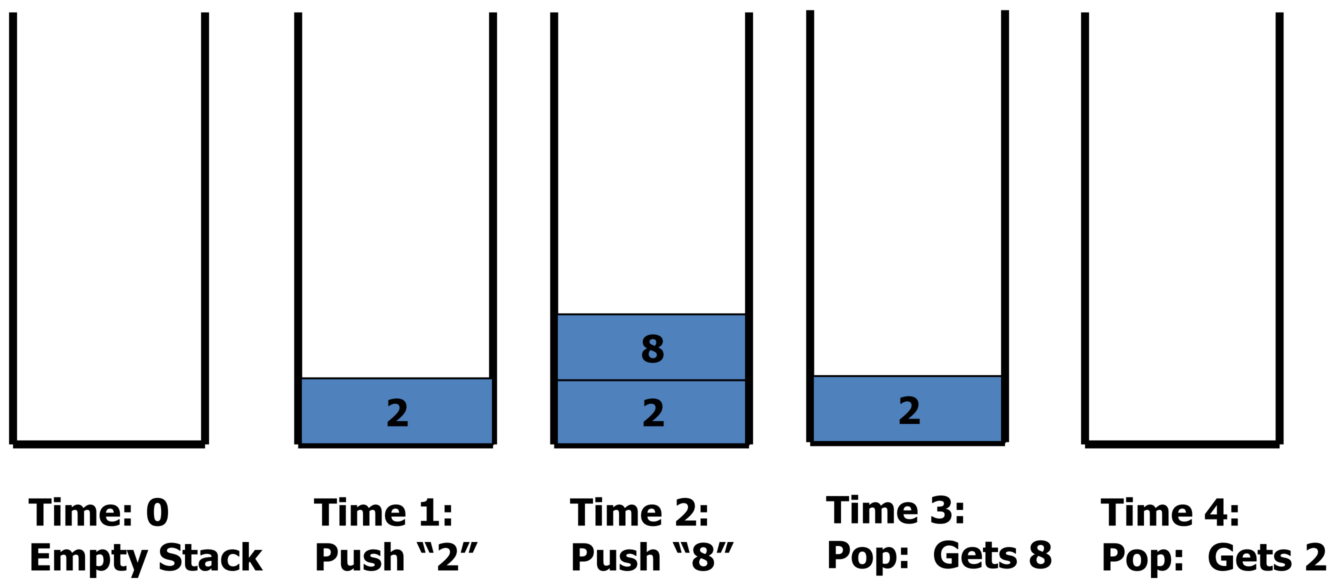
Stacks

- A stack is like a bunch of lunch trays in a cafeteria
- It has only two operations:
  - Push
    - You can push something onto the top of the stack
  - Pop
    - You can pop something off the top of the stack

- Let's see an example stack in action.

Stack Example

- The diagram below shows a stack over time
- We perform two pushes and two pops



Stack Details

- In computer science, a stack is a ***last in, first out*** (LIFO) data structure

- It can store any type of data, but has only two operations: push and pop
- Push adds to the top of the stack, hiding anything else on the stack
- Pop removes the top element from the stack

Stack Details

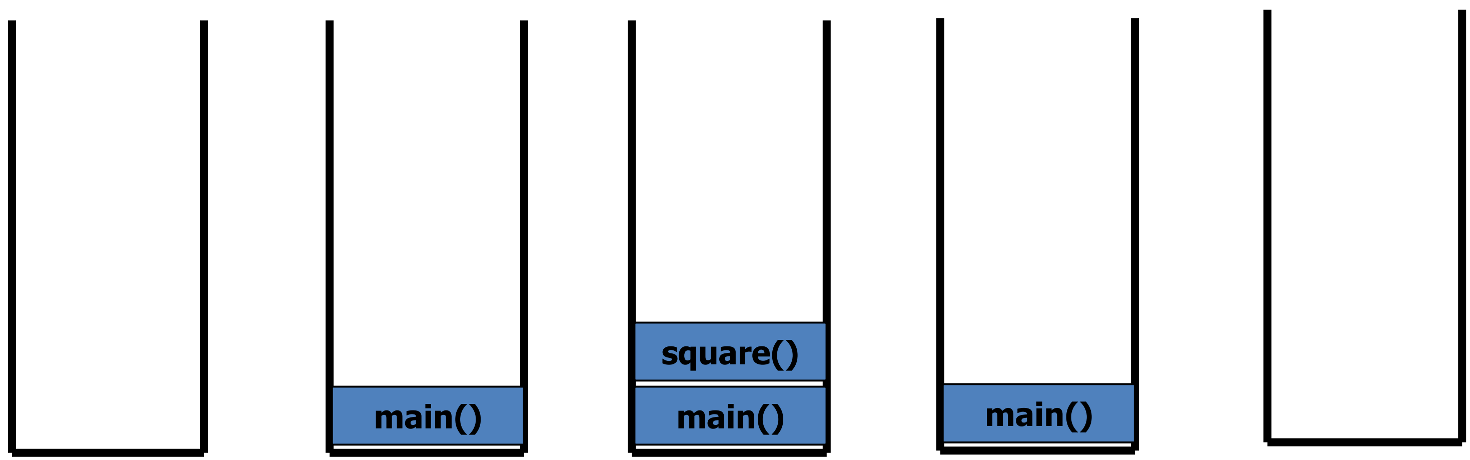
- The nature of the pop and push operations also means that stack elements have a natural order
- Elements are removed from the stack in the reverse order to the order of their addition
  - The lower elements are those that have been in the stack the longest

Stacks and Functions

- When you run your program, the computer creates a stack for you

- Each time you call a function, the function is pushed onto the top of the stack
- When the function returns or exits, the function is popped off the stack

Stacks and Functions Example



Time: 0
Empty Stack

Time 1:
Push: main()

Time 2:
Push: square()

Time 3:
Pop: square() returns a value. method exits.

Time 4:
Pop: main() returns a value. method exits.

Stacks and Recursion

- If a function calls itself recursively, you push another call to the function onto the stack

- We now have a simple way to visualize how recursion really works

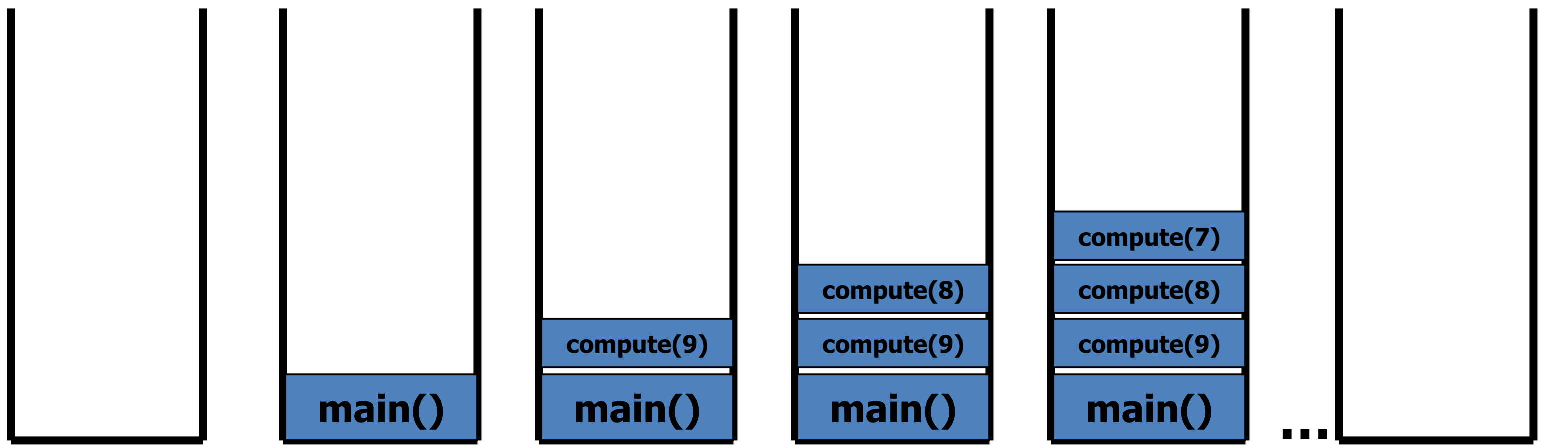
Toy Example of Recursion

```
def compute(intInput):
    print(intInput)
    if (intInput > 2):
        compute(intInput-1)

def main():
    compute(50)

main()
```

Here's the code again.

Now, that we understand stacks, we can visualize the recursion.

Stack and Recursion in Action



Time: 0 Empty Stack	Time 1: Push: main()	Time 2: Push: compute(9)	Time 3: Push: compute(8)	Time 4: Push: compute(7)	After returning from compute(2) pop everything

```
Inside compute(9):
print (intInput);         → 9
if (intInput > 2)
    compute(intInput-1);
```

```
Inside compute(8):
print (intInput);         → 8
if (intInput > 2)
    compute(intInput-1);
```

```
Inside compute(7):
print (intInput);         → 7
if (intInput > 2)
    compute(intInput-1);
```

Defining Recursion

“Cases” in Recursion

- A recursive function must have two things:

- At least one base case
  - When a result is returned (or the function ends)
  - “When to stop”
- At least one recursive case
  - When the function is called again with new inputs
  - “When to go (again)”

Terminology

```
def f(n):
  if n == 1:
    return 1
  else:
    return f(n - 1)
```

base case

recursive case

Recursion

```
def f(n):
  if n == 1:
   return 1
  else:
   return f(n + 1)
```

Find f(5)

We have a base case and a recursive case. What's wrong?

Recursion

The recursive case
should call the function
on a *simpler input*,
bringing us closer and closer
to the base case.

Recursion

```
def f(n):
   if n == 0:
    return 0
   else:
    return 1 + f(n - 1)

Find f(0)
Find f(1)
Find f(2)
Find f(100)
```

Recursion

```
def f(n):
   if n == 0:
    return 0
   else:
    return n + f(n - 1)

f(3)
3 + f(2)
3 + 2 + f(1)
3 + 2 + 1 + f(0)
3 + 2 + 1 + 0
6
```

Factorial

- $4! = 4 \times 3 \times 2 \times 1 = 24$

Factorial

- Does anyone know the value of 9! ?
- 362,880
- Does anyone know the value of 10! ?
- How did you know?

Factorial

- $9! = \quad 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$
- $10! = 10 \times \quad 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$
- $10! = 10 \times \quad 9!$
- $n! = n \times (n - 1)!$
- That's a recursive definition!

Factorial

```
def fact(n):
  return n * fact(n - 1)

fact(3)
3 * fact(2)
3 * 2 * fact(1)
3 * 2 * 1 * fact(0)
3 * 2 * 1 * 0 * fact(-1)
...
```

Factorial

- What did we do wrong?
- What is the base case for factorial?

Announcements

- Project 1 is/was due Wednesday

- Homework 8 is/was released Wednesday night
  - Last homework of the semester
  - Due the Wednesday before Thanksgiving
    - Plan ahead!